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Uses and Abuses of Multipliers in the Stand Prognosis Model

David A. Hamilton, Jr.

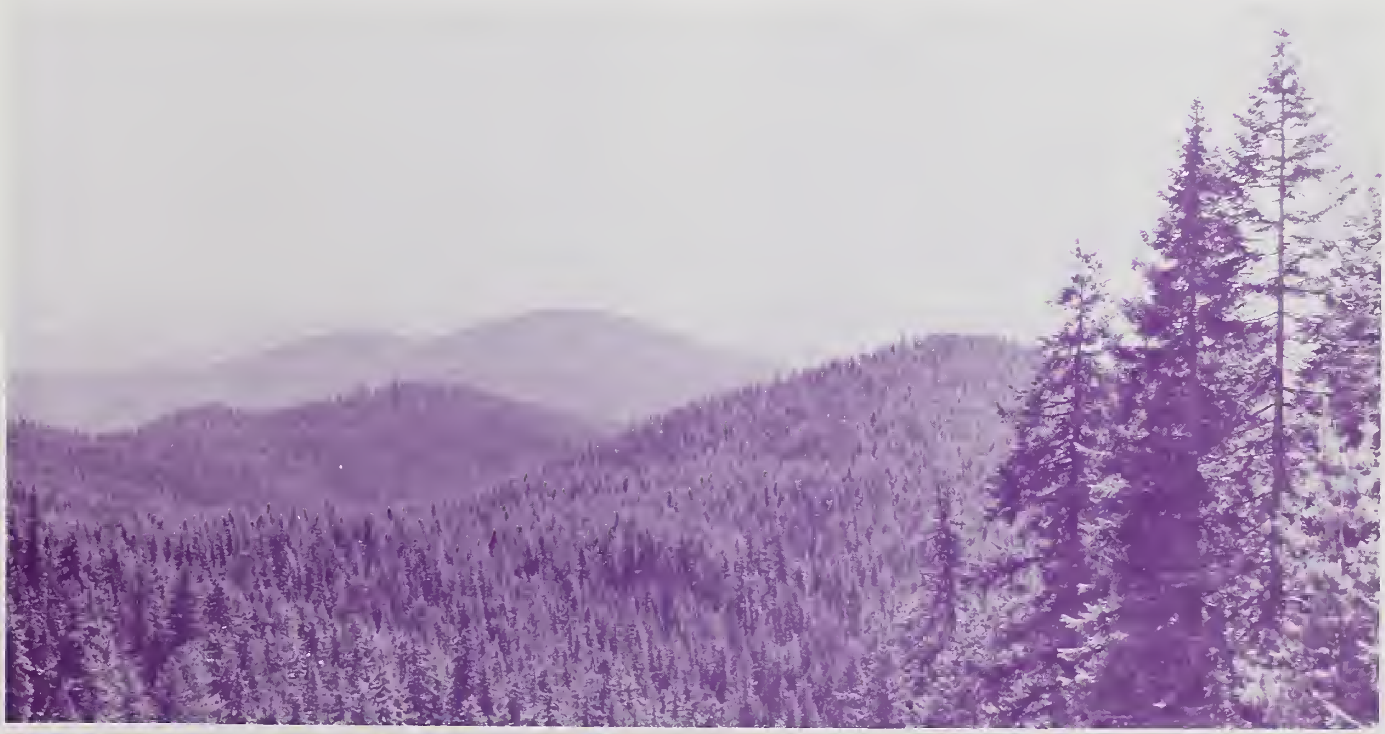
$$CR = f_e[CCF \cdot]$$

$$B = \frac{1}{\sqrt{2\pi}} \int_a^b x e^{-x^2/2} dx$$

$$\Delta H = f_N[\Delta D, HD] * HTGMULT$$

$$MORT = \frac{1}{1 + \exp[B_i X_i]} * MORTMULT$$

$$BAI = f_B[D.b.h., Habitat, Crown] * BAIMULT$$



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Research Summary

Multipliers that permit the user to tailor model performance for specific conditions are an important part of the Stand Prognosis Model. However, if multipliers are used incorrectly they can project patterns of stand development that are not biologically reasonable or, in more severe situations, they can cause the program to fail. Problems may arise either in selecting the proper set of multipliers to achieve a desired effect or in determining the appropriate value to assign to selected multipliers. In either case, the intuitively logical choice may not be correct. Examples that describe the impact of multipliers on simulated stand development and potential difficulties that can arise from the improper use of multipliers are discussed. Guidelines for the proper use of multipliers and suggested solutions to potential difficulties are presented. In general, it is incorrect to use both height increment and basal area increment multipliers to simulate a treatment that is expected to increase both height and diameter growth rates. Increases in both height and diameter growth rates can be simulated by applying either a height increment multiplier for small trees or a basal area increment multiplier for large trees. Detailed model output should be carefully examined any time extreme multipliers are used to simulate conditions that may not have been well represented in data used for model development.

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Uses and Abuses of Multipliers in the Stand Prognosis Model

David A. Hamilton, Jr.

The Stand Prognosis Model (Stage 1973; Wykoff 1986; Wykoff and others 1982) is an individual tree growth and yield model consisting of four primary component models: height increment, basal area increment, crown ratio, and mortality. Multipliers for component models have been a feature of most versions of the Stand Prognosis Model. The multipliers permit users to modify the performance of component models to simulate a desired effect. Unfortunately, the intuitively logical choice of multipliers is not always correct. Improper specification of multipliers can cause an effect to be counted twice or can push component models beyond their limits of extrapolation. Such problems may lead to biologically impossible patterns of growth (such as unrealistic height/diameter ratios) or in extreme cases can cause the program to fail because of computer run time errors (arguments of intrinsic FORTRAN functions get pushed outside permitted ranges).

This paper discusses examples of these problems in more detail and offers guidelines for determining the appropriate set of multipliers needed to simulate a desired effect. Methods for estimating the values to be assigned each selected multiplier are also suggested.

Version 6.1 of the Inland Empire variant includes four types of multipliers. Each of the four affects model performance somewhat differently, and each can be used to simulate somewhat different effects.

The first group of multipliers are automatically calculated calibration factors that scale height increment in trees less than 3 inches diameter breast height (d.b.h.) and basal area increment for trees 3 inches d.b.h. or larger. These multipliers are designed to modify model predictions of increment so they more nearly match any available increment observations for the site being projected. They provide a means of accounting for variation in increment due to causes that are not controlled in the Stand Prognosis Model (such as climate variation, variation within habitat types, or genetic improvement). The effect of calibration factors is attenuated toward an asymptote that is one-half the difference between the large tree calibration factor and one. They approach this asymptote by a factor of one-half the remaining difference in each succeeding projection cycle.

A second group of multipliers alter model components before calibration. These multipliers are input to the program with the keywords:

- READCORD—Modifies large tree basal area increment
- READCORR—Modifies small tree height increment
- READCORH—Modifies large tree height increment.

They provide a means of correcting any consistent bias in the performance of component models when the Stand Prognosis Model is applied to a specific set of stands. This could occur when extending the effective range of the model. If calibration suggests such a consistent bias, these multipliers can adjust component models to the "new" mean response level. Because these multipliers are applied before model calibration, individual stands still can be calibrated to account for stand-to-stand variability in expected model performance. These multipliers are not attenuated in succeeding projection cycles.

The third group includes three multipliers used to modify the performance of the regeneration establishment model. These multipliers are:

- HTADJ—Modifies regeneration tree heights
- STOCKADJ—Modifies the predicted probability of stocking
- SPECMULT—Modifies the probability of a species' occurrence.

The fourth group of multipliers are those applied to model components after calibration:

- HTGMULT—Modifies large tree height growth
- BAIMULT—Modifies large tree basal area growth
- MORTMULT—Modifies mortality rates
- REGDMULT—Modifies small tree diameter growth
- REGHMULT—Modifies small tree height growth.

For most variants of the Stand Prognosis Model (but not the Inland Empire variant), the change in crown ratio can also be modified with a multiplier (CRNMULT). These multipliers are not attenuated

over time. Instead, the user can specify their value for any projection cycle. They simulate effects such as silvicultural treatments or environmental changes that might be expected to affect a given stand but are not otherwise specifically included in the model. Examples include fertilization, planting genetically improved stock, and climate change. They also may be used to simulate the sensitivity of model performance to variation in individual component models.

The remainder of this paper will deal with the fourth group of multipliers. Parameters control how these multipliers are implemented. For the mortality multiplier, parameters control the species affected, the projection cycles for which the multiplier remains in effect, and the diameter range to which the multiplier applies. Parameters for the basal area increment and height increment multipliers are the same as for the mortality multiplier except that the diameter range cannot be specified.

For a sensitivity analysis of the Stand Prognosis Model, I modified all multipliers in the Inland Empire variant so different values could be specified for up to four diameter ranges in any specified projection cycle. This feature allowed the impact of modifications in overall model shapes on model output to be examined. The modified multipliers also proved essential to an analysis of stand level gains attributable to genetic improvement (Hamilton and Rehfeldt, in press) and were used in the analyses reported here. Although these modified multipliers are not currently available in version 6.1 of the Inland Empire variant, the author can help users needing to implement such code. In the remainder of this paper, Stand Prognosis Model refers to version 6.1 of the Inland Empire variant. However, most of the comments apply to other variants.

Relationships Between Multipliers and Component Models

Determining the proper value of a multiplier requires a thorough understanding of the component models that are affected and of interactions between component models. Basal area increment and height increment are modeled differently for trees less than 3 inches d.b.h. than for trees 3 inches d.b.h or larger. For trees less than 3 inches d.b.h., the height increment model is the primary model component; diameter increment is estimated by a height/diameter relationship. Height increment for small trees is a function both of tree height at the beginning of the projection period and of a set of other tree and stand characteristics (Wykoff 1986). Because of these relationships, a height increment multiplier affects both height increment and diameter increment, but a multiplier on d.b.h. increment has almost no effect on height increment.

For trees 3 inches d.b.h. or larger, the basal area increment model is the primary model component. For large trees, basal area increment is a function of d.b.h. and a set of individual tree and stand characteristics (but not height), while height increment is modeled as a function of height, d.b.h., d.b.h. increment, species, and habitat type. For trees 3 inches d.b.h. or larger, a basal area increment multiplier will affect both basal area increment and height increment, but a height increment multiplier will affect height increment with only a minor effect on d.b.h.

Crown ratio is a function of d.b.h., height, and other individual tree and stand characteristics, while mortality is a function of d.b.h. and other individual tree and stand characteristics, but not height. Thus, crown ratio will be affected by multipliers on either height increment or basal area increment, but mortality will only be affected by multipliers that affect d.b.h. and d.b.h. increment.

As indicated, a different model is used to predict height increment for small trees (less than 3 inches d.b.h.) and large trees (3 inches d.b.h. or larger). To provide a smooth transition between these models, trees between 2 and 10 inches d.b.h. (1 to 5 inches for lodgepole pine) use a weighted average of the estimated increments (Wykoff and others 1982). To achieve a desired change in height increment for trees in this diameter range, multipliers must be specified for both small tree and large tree height increment, even when all the trees are in either the large or small tree category.

Model Response to Multipliers

A series of examples illustrate how the Stand Prognosis Model responds to multipliers for component models. The results identify potential problems that can arise when the use of an apparently logical set of multipliers is not correct. They also provide the basis for determining the correct multiplier values to simulate a desired model outcome. Results of the examples are summarized in table 1. When interpreting these results, modelers need to remember that multipliers are applied to individual trees. The values used to evaluate multiplier impact are stand means that include the impact of mortality and other factors affecting growth. Thus, a multiplier of 1.2 seldom results in exactly a 20 percent increase in stand growth. Because in most of the examples in this paper we are dealing with young stands that do not have a wide diameter distribution, mortality is reasonably evenly distributed throughout the

Table 1—Impact of multipliers on performance of component models when applied at various times and in various combinations

Stand age during simulation	Multiplier				Change in diameter	Change in height
	Large tree height	Large tree diameter	Small tree height	Small tree diameter		
					- - - - - - - *Percent* - - - - - - -	
Age 30-40	1.2					16.2
	1.2		1.2			20.0
		1.2			17.2	13.4
	1.2	1.2			19.5	32.4
		1.2	1.2		19.6	17.1
Age 0-19			1.2		14.7	18.8
(3 model cycles)				1.2	10.7	–1.2
			1.2	1.2	24.0	16.0
	[1]1.2	1.2	1.2		18.9	20.9
Age 0-14				1.2	20.1	0.8
(2 model cycles),			1.2		22.6	22.3
(multipliers in second cycle only)			1.2	1.2	46.8	22.9

[1]Applied only to trees less than 3 inches d.b.h.

stand. This minimizes the effect of mortality on the evaluation of multiplier impact. To deal with the impact of random variation in Stand Prognosis Model output (Hamilton 1991), all results reported are based on the average of 20 replications of simulated stand development.

In each example, model response is demonstrated by observing the impact of assigning a value of 1.2 to: (1) the height increment multiplier, (2) the diameter or basal area increment multiplier, and (3) both multipliers. Results are most straightforward when the multipliers are applied to a stand made up of trees 3 inches d.b.h or larger. The following example simulates the growth for a single 10-year projection cycle (ages 30 through 40) of a ponderosa pine plantation established on a *Pseudotsuga menziesii*/*Physocarpus malvaceus* habitat type (Pfister and others 1977). When only a large tree height increment multiplier is applied, diameter increment is unaffected and height increment increases 16.2 percent. This response is as expected for diameter increment but seems to be too small for height increment. The discrepancy arises because height increment is a weighted average of the value predicted by the small tree equation (which is unaffected by the large tree multiplier) and that predicted by the large tree equation. If a multiplier is added for small tree height increment, the overall increase in height increment is raised to the intended 20.0 percent.

When only a large tree basal area increment multiplier is applied, diameter increment is increased 17.2 percent (19.4 percent on a basal area scale) and height increment is increased 13.4 percent. The following discussion examines how these results are affected by model structure and indicates how the impact on height increment can be expected to vary for different habitat types.

In the large tree basal area increment model, the multiplier is applied to change in squared diameter (*DDS*). Intuition might suggest that when a multiplier is applied to *DDS*, diameter growth will be changed by the square root of the multiplier. In this case, intuition would not be correct. Diameter growth for these trees is expressed as

$$DG = (dib^2 + DDS)^{1/2} - dib$$

where

DG = diameter increment
dib = diameter inside bark.

As the following equation shows, the effective multiplier (m^*) for diameter increment is not the square root of the multiplier on *DDS* but is instead a function of the multiplier (m), diameter inside bark at the start of the growth projection period, and *DDS*.

$$m^* = [(dib^2 + m \cdot DDS)^{1/2} - dib]/DG$$

Simulation shows that the effective multiplier is frequently only slightly less than the multiplier applied to *DDS* (in the example the effective multiplier on diameter increment is 1.172 while the multiplier applied to *DDS* is 1.2).

The increase of just 13.4 percent in height increment is explained by two factors. First, height increment is a weighted average of the large tree and small tree estimates for trees between 2 and 10 inches d.b.h.; in this example no multiplier has been applied to the small tree estimate. Second, the effect of a change in diameter growth on height increment for a single projection cycle is expressed by the term

$$b_3 \ln(DG)$$

The regression coefficient b_3, which varies by habitat type, controls how much of a percentage increase in diameter increment is passed on to the large tree estimate of height increment. For the example stand, $b_3 = 1.02372$. Therefore, when the effective multiplier for diameter increment is 1.172, the effective multiplier on the large tree component of estimated height increment is

$$e^{1.02372 \ln(1.172)} = 1.172^{1.02372} = 1.176$$

The proportion of the effect of a diameter increment multiplier that is passed on to height increment is smaller for all other habitat type classes. The proportion is smallest for habitat types *Abies lasiocarpa/Xerophyllum tenax*, *Tsuga mertensiana/Xerophyllum tenax*, and *Abies lasiocarpa/Vaccinium globulare*. For these habitat types, $b_3 = 0.34003$, and an effective multiplier of 1.172 for diameter increment results in a multiplier of 1.055 for height increment. In other words, a 17.2 percent increase in diameter increment translates into just a 5.5 percent increase in the large tree estimate of height increment.

When both height increment and basal area increment multipliers are applied to large trees, the increase in basal area increment is 19.5 percent, and the increase in height increment is 32.4 percent. The increase in height increment clearly demonstrates the double counting that occurs when multipliers are applied to both the large tree basal area increment model and the large tree height increment model.

For this example stand (which contains no trees less than 3 inches d.b.h.) a basal area increment multiplier for large trees and a height increment multiplier for small trees would simulate the increase in height and basal area increments without double counting. Setting each of these multipliers equal to 1.2 increases height increment by 17.1 percent and basal area increment by 19.6 percent. The increase in large tree basal area increment is not perfectly matched by the increase in height increment. Therefore, the multipliers required to achieve a specified increase in both height increment and basal area increment may best be determined through a systematic iterative search procedure that uses the Stand Prognosis Model to simulate the outcome of alternative sets of multipliers. This procedure will be discussed in more detail in the following section.

The effect of multipliers on projected development of stands made up of small trees is demonstrated by simulating the development of a ponderosa pine plantation from ages 0 through 19. Site conditions are assumed to be the same as they were in the previous example. Three projection periods are used (ages 0 to 8, 8 to 14, and 14 to 19). Multipliers were set at 1.2 and applied during all three projection periods.

When only a small tree height increment multiplier is applied, diameter increment increases 14.7 percent and height increment increases 18.8 percent. The effect of a change in small tree height increment on small tree diameter increment is estimated by the relationship

$$m^* = \frac{(H_1 + m \cdot HTG - 4.5)^{b_1} - (H_1 - 4.5)^{b_1}}{(H_1 + HTG - 4.5)^{b_1} - (H_1 - 4.5)^{b_1}}$$

where

HTG = height increment
H_1 = total height at beginning of prediction period
b_1 = regression coefficient.

For a 5-foot ponderosa pine with a height increment of 2 feet, a height increment multiplier of 1.2 results in an effective diameter increment multiplier of 1.2002. For other species, agreement between the two multipliers will be somewhat different (because b_i is species specific). Engelmann spruce and subalpine fir have the largest differences between the height increment and the effective diameter increment multipliers. For these species the effective diameter increment multiplier for the conditions just described would be 1.2164. Because the difference is still small, it should be expected that for all species there will be little practical difference in the two multipliers. These results hold for a single projection period. The 14.7 percent increase in diameter increment reported above is for three projection periods that included the time when many of the trees grew over the 3-inch d.b.h. boundary. The reduction from the expected 20 percent increase occurs because large tree basal area increment is not a function of height or height increment. None of the effect of the small tree height increment multiplier is passed on to the large tree component of diameter increment.

Using a small tree diameter increment multiplier as the only multiplier increases diameter increment 10.7 percent and decreases height increment 1.2 percent. This small decrease in height increment is likely due to a combination of factors. Changing diameter growth rates changes the timing of the transition from small tree to large tree models, changing the sequence of random numbers used by the stochastic portions of the models. The coefficients on the crown competition factor and on the

basal area in larger tree terms in the small tree height increment model are both negative. Thus, the increase in these two terms from the use of a small tree diameter increment multiplier should be expected to reduce predicted height increment.

The reduced impact of a small tree diameter increment multiplier on small tree diameter increment can be explained by model structure. For small trees, diameter increment is estimated as the difference between the diameter at the start of the projection period and at the end of the period. These two diameters are estimated by the height/diameter relationship described previously. The 10.7 percent increase was produced by a simulation that did not include a small tree height increment multiplier. Thus, the estimate of diameter increment for each projection period (before the multiplier is applied) is the same as would be obtained without a diameter multiplier. This eliminates any compounding of the impact of the small tree diameter increment multiplier from period to period.

Applying both small tree height increment and diameter increment multipliers results in a 24.0 percent increase in diameter increment but only a 16.0 percent increase in height increment. The result illustrates the reduction of height growth rates for small trees that appears to occur when a small tree diameter increment multiplier is used and the double counting of the increase in diameter increment when both small tree height increment and diameter increment multipliers are used.

Because the projection period for this simulation (ages 0 through 19 years) includes the period when trees are growing over the 3-inch d.b.h. boundary, a more appropriate set of multipliers would be a small tree height increment multiplier, a large tree height increment multiplier for trees less than 3 inches d.b.h. (which affects height increment through the weighted average discussed previously), and a large tree basal area increment multiplier. When each of these multipliers is set equal to 1.2 for the three projection periods, diameter increment increases 18.9 percent and height increment increases 20.9 percent.

By age 14 (the beginning of the final projection cycle) many trees have reached 3 inches d.b.h. Thus, the previous example includes both the effect of small tree multipliers on a stand of small trees and the impact of the stand growing past the 3-inch d.b.h. boundary. Further, it shows the effect of applying the multipliers over three growth periods. The final example in this section demonstrates the effects of applying small tree multipliers to stands made up solely of small trees. The same stand used in the previous example is used here. Stand development is simulated from ages 0 to 14. Multipliers are set to 1.2 but are applied only during the second growth period (ages 8 to 14). These conditions eliminate most of the factors suggested as causing the reduced responses to multipliers in the previous example.

Under these conditions a small tree diameter increment multiplier increases diameter increment by 20.1 percent and height increment by 0.8 percent. Applying just a small tree height increment multiplier increases height increment by 22.3 percent and diameter increment by 22.6 percent. Using both small tree diameter increment and height increment multipliers increases height increment by 22.9 percent and diameter increment by 46.8 percent. The large tree height increment multiplier for trees less than 3 inches d.b.h. has no effect in this case because all trees are less than 2 inches d.b.h. (the lower limit for using the weighted average to estimate height) at the start of the second projection cycle.

The double counting of diameter increment when both height increment and diameter increment multipliers are used does not vary from the previous example. However, for a stand composed entirely of trees less than 3 inches d.b.h., the effect of a height increment multiplier on height increment is closely matched by its effect on diameter increment.

Estimation of Multipliers

It is not unusual to conduct studies that examine the impact of some treatment on stand development for a period of 10 to 20 years. The results of such a study are often extrapolated over an entire rotation. Multipliers may be used to simulate the effect of changes in growth rates, observed on research study sites for limited time periods, when the changes are assumed to remain in effect throughout a rotation. Interdependencies among component models assure that an observed increase of X percent in height increment and a Y percent increase in basal area increment for an observed period cannot be simulated with multipliers of $1.X$ and $1.Y$ for height increment and basal area increment, respectively. The interdependencies also assure that the multipliers needed to achieve these simulated results over the observed period will not result in the same simulated percent changes at rotation age when the multipliers are allowed to remain in effect over the entire rotation. An example is the simulation of the effect of planting genetically improved stock on stand yield at rotation age. As suggested by Hamilton and Rehfeldt (in press), the proper multipliers can be estimated based on a systematic iterative search procedure in which the Stand Prognosis Model is used to simulate stand development with iteratively selected values of multipliers.

An important feature of the suggested approach is the inclusion of a step in which the effects of the particular stand being used in the estimation process

are accounted for. As long as there are no significant interactions between changes in growth response (represented by multipliers) and site factors, this step should increase the likelihood that the estimated multipliers are applicable across the existing variability in sites. If a significant interaction exists (if the relative response of genetically improved stock is different on different sites), the analysis suggested here must be repeated for any significantly different sites.

The estimation of multipliers needed to simulate the stand level gains expected from planting genetically improved ponderosa pine is used to demonstrate these procedures. Data from the Inland Empire Tree Improvement Cooperative was used to estimate gains in individual tree growth expected in the next generation of trees produced by a tree improvement program. At age 19 these estimates were 8.43 percent for height and 5.47 percent for diameter (Hamilton and Rehfeldt, in press). An intuitive choice for height increment and diameter increment multipliers in this case might be 1.0843 for height increment and 1.0547 for diameter increment. However, the discussion of component models given in a previous section and the results of the previous examples make clear that this choice would not have the intended effect. If these two multipliers were applied, either the anticipated increases in height increment or diameter increment would be double counted, depending on the tree's size at the beginning of the projection cycle.

For this example, the set of multipliers needed to simulate stand development includes a small tree height increment multiplier, a large tree diameter increment multiplier, and a large tree height increment multiplier applied only to trees less than 3 inches d.b.h. (it is used only in the large tree portion of the weighted average used to estimate height increment for trees less than 3 inches d.b.h.). Since data used in this example were available for only a few sites growing over a single time period under a single climatic regime, the impact of site and climate had to be removed from the estimation of multipliers. Two sets of multipliers were estimated: those needed to replicate the performance observed for unimproved stock and those needed to replicate anticipated performance of genetically improved stock. The multipliers needed to replicate the performance observed for unimproved stock account for the impact of uncontrolled site, climatic, and environmental factors on stand development. The multipliers needed to replicate anticipated performance of genetically improved stock account both for these uncontrolled site, climatic, and environmental factors and for the performance of the improved stock. The ratio of these two multipliers serves as an estimate of a multiplier representing only the impact of planting genetically improved stock.

Since the trees in this example had been measured three times from planting to age 19, variation in the change in growth rates attributable to genetic improvement could be determined over this time period. This required estimating the conditional multiplier for each projection period that was needed to achieve the expected height of genetically improved stock at the end of the period (given that the stand had already achieved the expected height in the previous period). To estimate these conditional multipliers directly would require data permitting the models to be calibrated for both improved and unimproved stock for each projection period (conditional on the stand having grown at the rate of improved stock in the previous period). Such data are probably not likely to be available for unimproved stock. Lack of such data in this example was overcome by the procedures described in the following paragraphs.

Large and small tree height increment multipliers needed to calibrate the model to achieve the desired height increment for the periods 0 to 8, 0 to 14, and 0 to 19 years were determined for both improved and unimproved stock (table 2). The ratio of these multipliers for each of the time periods, which estimates the multiplier representing only the impact of genetically improved stock, is presented in table 3. As indicated previously, this ratio eliminates the impact of uncontrolled site, climatic, or environmental variation from the multiplier. Basing this procedure on growth from age 0 to the end of each period removes the effects of uncontrolled variation without having to consider the confounding effect of differences in tree size between improved and unimproved stock at the beginning of each projection period.

Table 2—Multipliers for calibrating the Stand Prognosis Model for average stock and for genetically improved stock from age 0 to end of each period

		Random start			
	Multiplier[1]	1	2	3	4
Average stock	H8	1.1318	1.0382	1.0522	1.0186
	H14	0.9724	0.9500	0.9658	0.9400
	H19	0.9113	0.9000	0.9065	0.8911
Genetically improved stock	H8	1.3707	1.2571	1.2743	1.2340
	H14	1.0428	1.0128	1.0487	1.0090
	H19	0.9745	0.9634	0.9952	0.9638

[1]The multiplier label includes both the component model the multiplier is applied to and the ending year of the growth interval. For instance, H8 is the height increment multiplier that should be applied in the growth interval ending at year 8.

Table 3—Multipliers describing changes in height and diameter growth rates attributed to genetic improvement from age 0 to end of each period

Multiplier[1]	Random start 1	2	3	4
H8	1.2111	1.2108	1.2111	1.2115
H14	1.0724	1.0661	1.0858	1.0734
H19	1.0694	1.0704	1.0978	1.0816

[1]The multiplier label includes both the component model the multiplier is applied to and the ending year of the growth interval. For instance, H8 is the height increment multiplier that should be applied in the growth interval ending at year 8.

Expected height of genetically improved stock at the end of each projection period is determined by running the Stand Prognosis Model using the multipliers in table 3. These expected heights are used in the calculation of conditional multipliers. The conditional multiplier for the first period (ages 0 to 8) is the multiplier for this period in table 3. The conditional multiplier for ages 8 to 14 is the value needed to achieve the expected height at age 14 (given that the stand has grown with the appropriate multiplier from ages 0 to 8). Similarly, the multiplier for ages 14 to 19 is that needed to achieve the expected height at age 19 (given that the stand grew with the appropriate multipliers in effect from ages 0 to 8 and 8 to 14). Because height increment is a weighted average of large and small tree estimates, both small and large tree multipliers are needed for all trees.

For trees larger than 3 inches d.b.h., one further step is required to estimate the multipliers. After determining the set of large and small tree height increment multipliers needed to achieve the expected height, the large tree height increment multiplier is set equal to 1.0 for trees greater than 3 inches d.b.h. Then the large tree basal area increment multiplier needed to achieve the same expected height increment is determined. By substituting the basal area increment multiplier for the height increment multiplier in this way, the impact on height increment is unchanged and the height/diameter relationship assumed by the Stand Prognosis Model is not altered. The set of multipliers obtained by following these procedures is presented in table 4.

Table 4—Conditional multipliers describing changes in height and diameter growth rates attributed to genetic improvement for each period

Multiplier[1]	Random start 1	2	3	4	Average
H8	1.2111	1.2108	1.2111	1.2115	1.2111
H14	1.0237	1.0401	1.0485	1.0248	1.0343
H19	1.0793	1.0685	1.0993	1.0417	1.0722
D19	1.1023	1.0871	1.1274	1.0536	1.0926

[1]The multiplier label includes both the component model the multiplier is applied to and the ending year of the growth interval. For instance, H8 is the height-increment multiplier that should be applied in the growth interval ending at year 8.

Although estimates were available for expected gains in both height and diameter in genetically improved stock, the approach described has been concerned only with simulating anticipated gains in height increment. It is my opinion that unless there is strong evidence of a change in the height/diameter ratio, it is better to use only a single characteristic (preferably that measured with the highest precision) and assume that the underlying height/diameter ratio used by the model will remain valid. For these data and for trees of this size, height is the characteristic measured with the most confidence. Graphical examination of the data produced no strong evidence for a change in height/diameter ratio.

Dangers of Using Very Small Basal Area Multipliers

Multipliers may be used to extrapolate or calibrate models so they simulate stand development under treatments or site conditions not included in the data used to fit individual component models. However, there are limits (mostly undefined) beyond which model extrapolation may lead to problems. The following example presents a case in which the model failed when a set of multipliers was used that logic suggested should have simulated possible conditions.

As part of a sensitivity analysis of the Stand Prognosis Model, I examined its sensitivity to variation in levels of the four primary component models. The extremes of the variation examined were represented by multipliers of 0.1 and 1.9 for each component model. In all likelihood multipliers of this size pushed the component models outside the range of values for which data were available in model development. However, one of the objectives of a sensitivity analysis is to determine how a model performs when it is applied under such conditions. Further, such multipliers would be appropriate for some conditions. For example, a basal area increment multiplier of 0.1 would seem appropriate for simulating development of a stagnated stand.

When a basal area increment multiplier of 0.1 and a mortality multiplier of 1.9 were applied to a mixed-species stand on the St. Joe National Forest, problems arose. The most obvious problems were run time errors that occurred when parameters of intrinsic FORTRAN functions were pushed outside their permitted range. Investigation revealed that the

FORTRAN error was caused by very large simulated individual tree heights (greater than 1,600 feet). Further study indicated that problems existed even when run time errors were not generated. If the mortality multiplier in this example was reduced to 1.4, run time errors disappeared. However, tree heights greater than 1,600 feet were still being simulated. Reducing the mortality multiplier to 1.0 led to simulated heights greater than 500 feet. If the basal area increment multiplier was increased to 0.3, the pattern of height increment became "normal" throughout the rotation, regardless of the values assigned to other multipliers. I conclude that using the basal area increment multiplier to simulate stand stagnation may lead to biologically implausible results. It would probably be wise to carefully examine detailed model output any time extreme multipliers are used to simulate conditions that may not have been included in data used when the model was developed.

The previous problem arose because the small tree height increment model does not contain a term to bound height growth. Under normal levels of diameter increment this would not be a problem. However, if diameter growth rates are essentially stopped when the tree is close to 3 inches d.b.h., a long simulation (200 years in the example) can lead to the problem. The small tree component of the weighted height increment estimate increases, slowly at first but more rapidly as the stand progresses through the rotation. The small tree height increment estimate is strongly influenced by tree height. The only impact of diameter growth on the small tree height increment model is through a density term. Since individual tree diameters are not increasing, density is not increasing. The result is that the small tree estimate of height increment increases without bound. In the absence of diameter increment, which would lead to progressively smaller weights for the small tree height increment estimate, the overall weighted estimate of tree height increment also increases without bound.

The problem is exaggerated by the way the large tree height increment model deals with tall, thin trees. In this model component the regression coefficient for height is positive and the coefficient for d.b.h. is negative. As a result, tall, thin trees get a greater than expected increase in estimated height increment from the height term and a smaller than expected decrease in estimated height increment from the d.b.h. term. Thus, estimated height increment is larger than would be expected if the tree had a "normal" height/diameter relationship. The height squared term in the large tree height increment model (with a negative regression coefficient) keeps the large tree portion of the weighted estimate of height increment from increasing without bound. However, height increment is accelerated under the conditions and during the critical time periods when excessive estimates of small tree height increment pushed the process out of control in this example.

Discussion and Conclusions

The proper set of multipliers needed to simulate any given or anticipated result may be unique to the particular situation. The selection may be based on the information available, the desired result, and the component models involved. Ultimately, both the determination of the proper set of multipliers and the estimation of the appropriate values for the multipliers must be based on a thorough understanding of the component models and their interactions with each other. While the set of multipliers needed in any given situation may be unique, some generalizations can be made. When the objective is to simulate the impact of a factor that changes overall stand growth rates (such as fertilization, irrigation, genetic improvement, and so forth), a standard set of multipliers may be suggested. These include a small tree height increment multiplier, a large tree height increment multiplier applied only to trees less than 3 inches d.b.h., and a large tree basal area increment multiplier. This set of multipliers changes growth rates of all individual trees but assumes that the underlying height/diameter relationships built into the component models are not modified. When the objective is to simulate the impact of changes in height/diameter ratios, either diameter increment multipliers for small trees or height increment multipliers for large trees (or both) could be added to the required set of multipliers.

If the multiplier is to simulate the impact of an *X* percent change in a single model component, the multiplier should be set equal to 1.*X*. However, when the objective is to determine the proper value of multipliers needed to simulate some observed phenomena (such as an expected *Y* percent increase in height at age 19 in a plantation of genetically improved stock), the most appropriate method is to use a systematic iterative search procedure in which the Stand Prognosis Model is used to simulate stand development for iteratively selected sets of multipliers. The iterative search procedure is also needed when the treatment is to be applied on different sites, when the treatment effect is expected to continue beyond the time span for which research results are available, or when the treatment effect must be expressed by changes in several of the interdependent model components.

In most cases it is incorrect to use both a height increment and a basal area increment multiplier to simulate a treatment effect that is expected to

increase both height and diameter growth rates. Applying either a height increment or basal area increment multiplier increases both height and diameter growth rates. The determination of the appropriate multiplier depends on the size of trees that make up the stand. As a general rule, it is most appropriate to use the multiplier or multipliers that correspond to the primary submodel (that is, the height increment multiplier for small trees and the basal area multiplier for large trees). Only when the height/diameter ratio needs to be changed should both a height increment and a basal area increment multiplier be used when simulating stand development.

Finally, care must be taken in applying multipliers that result in very large changes in the component models. Such multipliers may push model components beyond the range of data used in model development; the resulting extrapolation may lead to unexpected and unrealistic model performance. Detailed model output should be carefully examined any time extreme multipliers are used to simulate conditions that may not have been well represented in data used for model development.

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Users of the Stand Prognosis Model may have difficulties in selecting the proper set of multipliers to simulate a desired effect or in determining the appropriate value to assign to selected multipliers. A series of examples describe impact of multipliers on simulated stand development. Guidelines for the proper use of multipliers are presented.

Keywords: stand development, growth models, silviculture, simulation models

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